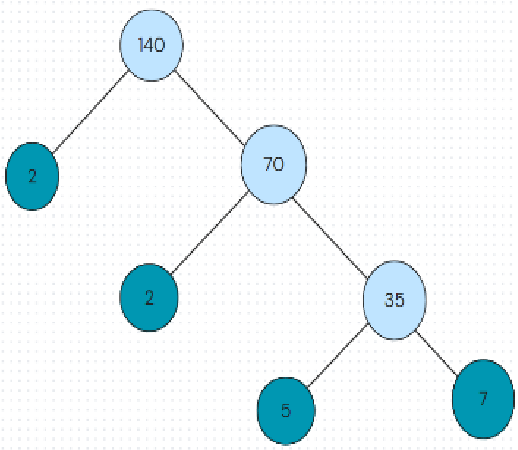
EXERCISE 1.1

1. Express each number as a product of its prime factors:

(i) 140

Sol.



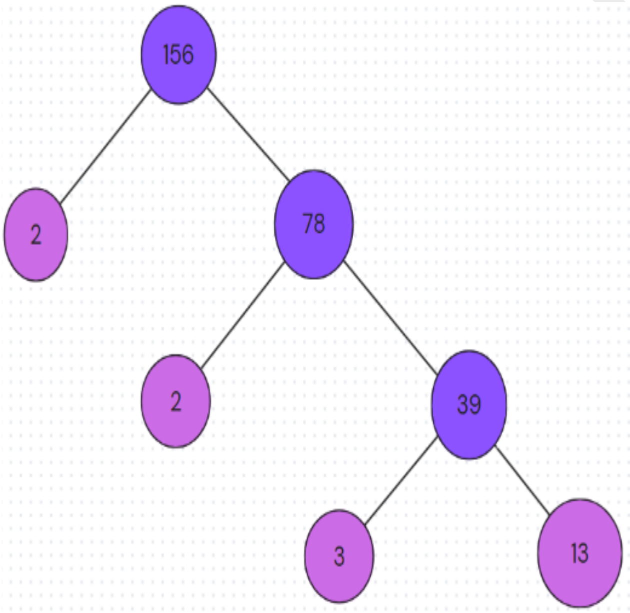
2	140
2	70
5	35
7	7
	1

$\therefore 140 = 2 \times 2 \times 5 \times 7$
$\quad = 2^2 \times 5 \times 7$ Ans.

$\therefore 140 = 2 \times 2 \times 5 \times 7$
$\quad = 2^2 \times 5 \times 7$ Ans.

(ii) 156

Sol.



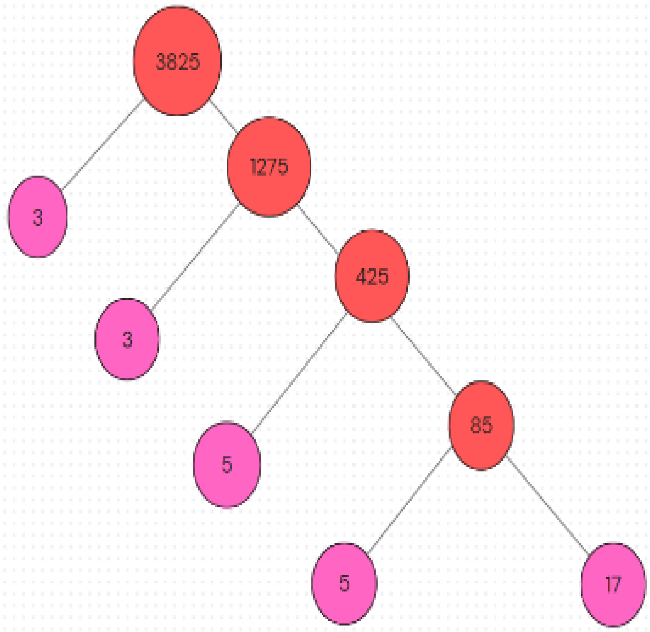
2	156
2	78
3	39
13	13
	1

$\therefore 156 = 2 \times 2 \times 3 \times 13$
$\quad = 2^2 \times 3 \times 13$ Ans.

$\therefore 156 = 2 \times 2 \times 3 \times 13$
$\quad = 2^2 \times 3 \times 13$ Ans.

(iii) 3825

Sol.



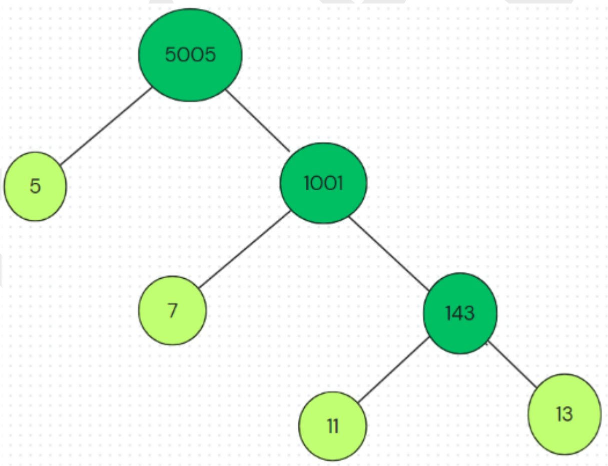
3	3825
3	1275
5	425
5	85
17	17
	1

$\therefore 3825 = 3 \times 3 \times 5 \times 5 \times 17$
$= 3^2 \times 5^2 \times 17$ Ans.

$\therefore 3825 = 3 \times 3 \times 5 \times 5 \times 17$
$= 3^2 \times 5^2 \times 17$ Ans.

(iv) 5005

Sol.



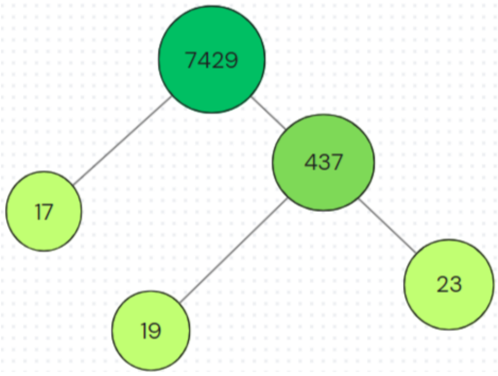
5	5005
7	1001
11	143
13	13
	1

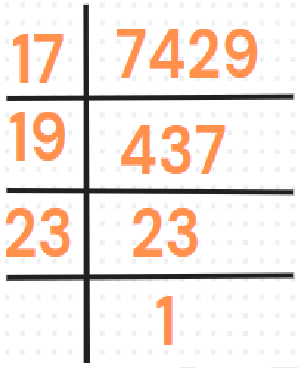
$\therefore 5005 = 5 \times 7 \times 11 \times 13$ Ans.

$\therefore 5005 = 5 \times 7 \times 11 \times 13$ Ans.

(v) 7429

Sol.





$\therefore$ 7429 = 17 $\times$ 19 $\times$ 23 Ans.

$\therefore$ 7429 = 17 $\times$ 19 $\times$ 23 Ans.

2. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.

(i) 26 and 91

Sol. 26 = 2 $\times$ 13

91 = 7 $\times$ 13

$\therefore$ H. C. F. = 13

And L. C. M. = 2 $\times$ 7 $\times$ 13

= 182 Ans.

Now, LCM × HCF = Product of the two numbers

$\Rightarrow$ 182 × 13 = 26 × 91

$\Rightarrow$ 2366 = 2366 is true

$\therefore$ LCM × HCF = Product of the two numbers. Verified.

Alternate method: -

26 = $2^1 \times 13^1$

91 = $7^1 \times 13^1$

$\therefore$ H. C. F. = 13

And L. C. M. = $2^1 \times 7^1 \times 13^1$

= 182 Ans.

Now, LCM × HCF = Product of the two numbers

$\Rightarrow$ 182 × 13 = 26 × 91

$\Rightarrow$ 2366 = 2366 is true

$\therefore$ LCM × HCF = Product of the two numbers. Verified.

(ii) 510 and 92

Sol. $510 = 2 \times 3 \times 5 \times 17$

$92 = 2 \times 2 \times 23$

$\therefore$ H. C. F. = 2

And L. C. M. = $2 \times 3 \times 5 \times 17 \times 2 \times 23$

= 23460

Now, LCM × HCF = Product of the two numbers

$\Rightarrow$ 23460 × 2 = 510 × 92

$\Rightarrow$ 46920 = 46920 is true

$\therefore$ LCM × HCF = Product of the two numbers. Verified.

Alternate method: -

$510 = 2^1 \times 3^1 \times 5^1 \times 17^1$

and $92 = 2^2 \times 23^1$

$\therefore$ H. C. F. = 2^1 = 2

And L. C. M. = $2^1 \times 7^1 \times 13^1$

= 182 Ans.

Now, LCM × HCF = Product of the two numbers

$\Rightarrow$ 23460 × 2 = 510 × 92

$\Rightarrow$ 46920 = 46920 is true

$\therefore$ LCM × HCF = Product of the two numbers. Verified.

(iii) 336 and 54

Sol. $336 = 2 \times 2 \times 2 \times 3 \times 2 \times 7$

$54 = 2 \times 3 \times 3 \times 3$

$\therefore$ H. C. F. = 2×3

= 6

And L. C. M. = $2 \times 3 \times 2 \times 2 \times 2 \times 7 \times 3 \times 3$

= 3024

Now, LCM × HCF = Product of the two numbers

$\Rightarrow$ 3024 × 6 = 336 × 54

$\Rightarrow$ 18144 = 18144 is true

$\therefore$ LCM × HCF = Product of the two numbers. Verified.

Alternate method: -

$336 = 2^4 \times 3^1 \times 7^1$

and $54 = 2^1 \times 3^3$

$\therefore$ H. C. F. = $2^1 \times 3^1$

= 6

And L. C. M. = $2^4 \times 3^3 \times 7^1$

= $16 \times 27 \times 7$

= 3024 Ans.

Now, LCM × HCF = Product of the two numbers

⇒ 3024 × 6 = 336 × 54

⇒ 18144 = 18144 is true

∴ LCM × HCF = Product of the two numbers. Verified.

3. Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21

Sol. 12 = $3 \times 2 \times 2$

15 = 3×5

And 21 = 3×7

∴ H. C. F. = 3

And L. C. M. = $3 \times 2 \times 2 \times 5 \times 7$

= 420

∴ LCM= 420 and HCF = 3 Ans.

Alternate method: -

12 = $3^1 \times 2^2$

15 = $3^1 \times 5^1$

and 21 = $3^1 \times 7^1$

∴ H. C. F. = 3^1 = 3

And L. C. M. = $3^1 \times 2^2 \times 5^1 \times 7^1$

= $3 \times 4 \times 5 \times 7$

= 420 Ans.

∴ LCM= 420 and HCF = 3 Ans.

(ii) 17, 23 and 29

Sol. 17 = 1×17

23 = 1×23

And 29 = 1×29

∴ H. C. F. = 1

And L. C. M. = $17 \times 23 \times 29$

= 11339

∴ LCM= 11339 and HCF = 1 Ans.

Alternate method: -

$17 = 1^1 \times 17^2$

$23 = 1^1 \times 23^1$

and $29 = 1^1 \times 29^1$

$\therefore$ H. C. F. $= 1^1 = 1$

And L. C. M. $= 17^1 \times 23^1 \times 29^1$

$= 17 \times 23 \times 29$

$= 11339$ Ans.

$\therefore$ LCM= 11339 and HCF = 1 Ans.

(iii) 8, 9 and 25

Sol. $8 = 1 \times 2 \times 2 \times 2$

$9 = 1 \times 3 \times 3$

And $25 = 1 \times 5 \times 5$

$\therefore$ H. C. F. = 1

And L. C. M. $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$

$= 1800$

$\therefore$ LCM= 1800 and HCF = 1 Ans.

Alternate method: -

$8 = 1^1 \times 2^3$

$9 = 1^1 \times 3^2$

And $25 = 1^1 \times 5^2$

$\therefore$ H. C. F. $= 1^1 = 1$

And L. C. M. $= 2^3 \times 3^2 \times 5^2$

$= 8 \times 9 \times 25$

$= 1800$ Ans.

$\therefore$ LCM= 1800 and HCF = 1 Ans.

4. Given that HCF (306, 657) = 9, find LCM (306, 657).

Sol. The given numbers are 306 and 657.

And H.C.F. = 9

We know, LCM × HCF = Product of the two numbers

$\Rightarrow$ LCM × 9 = 306 × 657

$\Rightarrow \text{LCM} = \frac{306 \times 657}{9}$

$\therefore$ LCM = 22338. Ans.

5. Check whether 6^n can end with the digit 0 for any natural number *n*.

Sol. 6^n end with 0 if 5 and 2 are the primes of 6.
But primes of 6 are 2 and 3.
Since 5 is not a prime of 6
$\therefore$ 6^n Cannot end with the digit 0. Ans.

6. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Sol. $7 \times 11 \times 13 + 13$
$= 13(7 \times 11 + 1)$
$= 13(77 + 1)$
$= 13 \times 78$
$= 13 \times 13 \times 3 \times 2$
It has more than 2 factors.
$\therefore$ It is a composite number.

(ii) $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$
$= 5(7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$
$= 5(1008 + 1)$
$= 5 \times 1009$
$= 5 \times 1009 \times 1$
It has more than 2 factors.
$\therefore$ It is a composite number.

7. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Sol. Time taken by Sonia to drive one round = 18 minutes
And, time taken by Ravi to drive one round = 12 minutes
$\therefore$ Time taken by both to meet again = LCM of 18 and 12
= 36 minutes

$18 = 2^1 \times 3^2$
$12 = 2^2 \times 3^1$
$\therefore$ L. C. M. $= 2^2 \times 3^2$
$= 4 \times 9$
$= 36$

$\therefore$ After 36 minutes they will meet again at the starting point. Ans.

EXERCISE 1.2

1. Prove that $\sqrt{5}$ is irrational.

Sol. Let $\sqrt{5}$ is rational.

$\Rightarrow \sqrt{5} = \frac{p}{q}$, where p and q are co-prime integers.

$\Rightarrow q\sqrt{5} = p$

Squaring both sides, we have

$\Rightarrow q^2 5 = p^2$

$\Rightarrow p^2$ is divisible by 5

$\Rightarrow p$ is also divisible by 5………………… (i)

Let $p = 5\,m$

$\Rightarrow 5\,q^2 = 25\,m^2$

$\Rightarrow q^2 = 5\,m^2$

$\Rightarrow q^2$ is divisible by 5

$\Rightarrow q$ is also divisible by 5…………………. (i)

From (i) and (ii), we have

p and q are divisible by 5

But p and q are co-prime.

Which is a contradiction.

∴ Our supposition is wrong.

∴ $\sqrt{5}$ is irrational. Proved.

2. Prove that 3 + 2 $\sqrt{5}$ is irrational.

Sol. Let 3 + 2 $\sqrt{5}$ is rational.

$\Rightarrow$ 3 + 2 $\sqrt{5} = \frac{p}{q}$, where p and q are co-prime integers.

$\Rightarrow$ 2 $\sqrt{5} = \frac{p}{q}$ - 3

$\Rightarrow \sqrt{5} = \frac{p - 3q}{2q}$

Here p and q are integers, so $\frac{p-3q}{2q}$ is rational

$\sqrt{5}$ is also rational.

Which contradict the fact that $\sqrt{5}$ is irrational.

∴ Our supposition is wrong.

∴ $3 + 2\sqrt{5}$ is irrational. Proved.

3. Prove that the following are irrationals:

(i) $\frac{1}{\sqrt{2}}$

Sol. Let $\frac{1}{\sqrt{2}}$ is rational.

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{p}{q}$, where p and q are co-prime integers.

$\Rightarrow \sqrt{2} = \frac{q}{p}$

Here p and q are integers, so $\frac{q}{p}$ is rational

$\sqrt{2}$ is also rational.

Which contradict the fact that $\sqrt{2}$ is irrational.

$\therefore$ Our supposition is wrong.

$\therefore \frac{1}{\sqrt{2}}$ is irrational. Proved.

(ii) 7 $\sqrt{5}$

Sol. Let $7\sqrt{5}$ is rational.

$\Rightarrow 7\sqrt{5} = \frac{p}{q}$, where p and q are co-prime integers.

$\Rightarrow \sqrt{5} = \frac{p}{7\,q}$

Here p and q are integers, so $\frac{p}{7\,q}$ is rational

$\sqrt{5}$ is also rational.

Which contradict the fact that $\sqrt{5}$ is irrational.

$\therefore$ Our supposition is wrong.

$\therefore 7\sqrt{5}$ is irrational. Proved.

(iii) 6 + $\sqrt{2}$

Sol. Let 6 + $\sqrt{2}$ is rational.

$\Rightarrow$ 6 + $\sqrt{2} = \frac{p}{q}$, where p and q are co-prime integers.

$\Rightarrow \sqrt{2} = \frac{p}{q} - 6$

$\Rightarrow \sqrt{2} = \frac{p - 6q}{q}$

Here p and q are integers, so $\frac{p - 6q}{q}$ is rational

$\sqrt{2}$ is also rational.

Which contradict the fact that $\sqrt{2}$ is irrational.

$\therefore$ Our supposition is wrong.

$\therefore 6 + \sqrt{2}$ is irrational. Proved.